ABSTRACT

Objective: To develop a search coil with a capacity-resistor (CR) integration circuit for instantaneous measurement of a pulsed magnetic field, and to compare the magnetic field structure measured by the novel device with the theoretical values.

Methods: The integration circuit consists of a capacitor and a resistor connected in series, and generates a voltage output across the capacitor. For magnetic field measurement, we selected capacitance × resistance = 4 ms. Using the search coil and CR integration circuit, we measured the magnetic flux density generated by a circular coil, over a range of magnetic flux densities (0.1 to 1.4 T) and pulse widths (0.1 to 0.6 ms). We also obtained the corresponding magnetic fields by numerical integration of the electromotive force induced in the search coil. Then we measured the distribution of magnetic flux densities generated by a commercial figure-of-eight coil commonly used in transcranial magnetic stimulation (TMS), and compared the results with the magnetic field derived from model calculation.

Results: The values measured by the CR circuit and the values obtained by numerical integration showed good agreement (R² = 0.9993). At 100% output from the stimulator, the peak magnetic flux density of the horizontal and vertical magnetic fields at a distance 10-30 mm from the figure-of-eight coil surface was 0.2 to 0.5 T. The measured magnetic field showed less focalization when compared with the values obtained from model calculation.

Conclusions: We identified the appropriate CR combination for the integration circuit used to measure a pulsed magnetic field. This device would be useful for quantitative analysis of TMS-induced effects on cortical function.

Key words: transcranial magnetic stimulation, magnetic field structure, figure-of-eight coil, integration circuit, teslameter, magnetic flux density
Introduction

Transcranial magnetic stimulation (TMS) noninvasively excites neurons in the cerebral cortex by inducing weak electric currents (eddy currents) [1, 2]. Repetitive TMS (rTMS) has recently attracted considerable attention as an effective method to induce plastic changes in the central nervous system. For example, rTMS has been applied for the restoration of motor function following hemiplegia [3–7] and modulation of cognitive function [8–11].

A figure-of-eight (FOE) coil is widely used for medical purposes [12, 13]. The FOE coil is composed of two loops approximately 70 mm in diameter, through which an electric current flows in opposite directions to produce monophasic or biphasic pulsed stimulation. Although the electric field properties of commercial FOE coils used in TMS have been reported [14], the actual eddy currents induced in the human brain have not been measured. Furthermore, these currents are difficult to estimate because of the complicated brain structure and insufficient information about the electromagnetic properties, such as conductivity and permittivity, of the human brain.

While actual eddy currents cannot be measured, it is essential to know the electromagnetic characteristics such as the magnetic field intensity of individual TMS coils for quality control of TMS studies. Previously, we have found that the coefficient of variation for the pulsed magnetic field intensity measured by commercially available gauss meters for static or pulsed magnetic field intensity measured by commercial FOE coils used in TMS have been reported [14], the actual eddy currents induced in the human brain have not been measured. Furthermore, these currents are difficult to estimate because of the complicated brain structure and insufficient information about the electromagnetic properties, such as conductivity and permittivity, of the human brain.

Although pulsed magnetic field intensity can be estimated by numerical integration of the electromotive force induced in the search coil, the calculation takes time and effort. Thus, we aimed to develop a search coil with a capacitor-resistor (CR) integration circuit to measure the pulsed magnetic field instantaneously. The CR integration circuit, in which the capacitor and resistor are connected in series, generates an output across the capacitor that approximates the numerical integration when the time constant (capacitance x resistor) is sufficiently large compared with the pulse width. We examined the accuracy of the novel instrument by comparing the measured results with those obtained by numerical integration of the electromotive force, and demonstrated its usefulness by comparing the measured and theoretical magnetic field structure induced by a commercially available FOE coil.

Methods

1. Development of an instrument to measure pulsed magnetic field intensity

When a magnetic pulse passes through a search coil, a voltage V is induced that can be expressed by equation (1).

\[ V = nS \frac{dB}{dt} \]  

(1)

where n is the number of turns of the search coil, S is the surface area of the coil, B is the magnetic flux density, and t is time.

To express the magnetic flux density B, equation (2) is derived by integration of equation (1).

\[ B = \frac{1}{nS} \int V dt \]  

(2)

The strength of the magnetic field can be calculated theoretically from (2) by numerical integration of the induced V. This calculation, however, is fairly time-consuming because of the large amount of data, and thus is not applicable to real-time measurements. In order to avoid such complicated numerical integration, the time integration term of equation (2) is replaced by a CR integration circuit (Fig. 1). In this circuit, a resistor with resistance R and a capacitor with capacitance C are connected to the search coil. When the search coil generates a voltage V, and a current I flows in R for a time T, then the stored electric charge Q in the capacitor is:

\[ Q = CV_c(T) = \int_0^T i(t) dt = \int_0^T \frac{V(t) - V_c(t)}{R} dt \]

where \( V_c \) is the charge voltage of the capacitor.

In this equation, we assume that CR >> T, and \( V_c(0) = 0 \).

Since the time integration in equation (2) has been replaced by the time constant CR, B is calculated by the simple equation (3).

\[ B = \frac{1}{nS} V_c CR \]

(3)

Since equation (3) is so simple and there is no criterion for the values of C and R, it is important to select appropriate C and R values for practical measurements of B. Thus, we investigated the relationship between CR values and CR integral magnetic flux density.

Figure 1. Schematic representation of the CR integration circuit with a search coil. The capacitor (C) and resistor (R) are connected in series. The output of the circuit is voltage across the capacitor. The search coil is placed in the magnetic field to be measured.

A pulsed magnetic field with a pulse width of 0.17 ms was generated by a custom-made circular coil, and measured using the search coil used in the present study. The search coil was positioned coaxially just above the excitation coil. Numerical integration showed a magnetic flux density amplitude of 0.591 T. As the induced V is low (around 1 V), we selected R lower than 5 kΩ, and measured its resistance up to 4 digits. In addition, we used a ceramic capacitor with a high Q value and measured its capacitance up to 4 digits at 1 kHz. We then set up a CR integration Gauss meter using these criteria. CR integration was performed at constant capacitance (9.705 μF at 1 kHz alternating current) and various resistances ranging from 30 to 2201 Ω. Five measurements for each resistance value were made, and the mean and standard deviation were calculated.

Based on experimental results as shown below (1. Relationship between CR values and CR integral of the search coil was oriented parallel to the respective coordinate. Five measurements were taken and averaged at each site.

2. Measurement of pulsed magnetic field generated from a FOE coil

We measured pulsed magnetic flux density amplitudes generated by a commercially available 70-mm FOE coil (9925-00, Magstim, Carmarthenshire, UK) and a stimulator (MRS 1000/50, Magstim). The handle of the FOE coil was held horizontally with its flat side upward by a fastening device. The center of the FOE coil surface was the origin of the x, y and z coordinates (0, 0, 0). The x-axis was the long axis of the FOE coil; the y-axis was perpendicular to the x-axis; and the z-axis was the vertical axis. By moving the search coil by 5-mm increments in the x- and y-directions and 10-mm in the z-direction in a 3-D coordinate system, we measured magnetic flux densities at a total of 1575 sites (25×21×3).

3. Numerical calculation of magnetic flux density

We numerically calculated the spatial distribution of the magnetic flux density, using the FOE coil model. As shown in Fig. 3, the model uses a pair of current loops with 9 turns each [15] and center at A (x0, 0, 0) and B (−x0, 0, 0) on the x-axis. The outermost loops contact each other at the coordinate origin O. For convenience, the loops are referred to as $C_j$ ($j = 1, 2, \ldots, 9$), where $j$ indicates loop number, and ± represents positive and negative regions of the x-axis. The electric current in $C_j$ flows in the opposite direction to that in $C_j$ for all $j$. The loops are excited by an alternating electric current, $I = I_0 e^{-\omega t}$, where $I_0$ is the peak current, and $\omega = 2\pi f$ is the angular frequency. In this study, we focus on the field distribution near the coil where the radiation effect is negligible, so that the electromagnetic fields can be calculated simply in an electrostatic manner. The nontrivial components of the vector potentials for the FOE coil are given by

$$A_x = \sum_{j=1}^{9} (A_{x,j} + A_{x,j}^\prime)$$

$$A_y = \sum_{j=1}^{9} (A_{y,j} + A_{y,j}^\prime)$$

where

$$A_{x,j} = \frac{\mu_0 I_0}{4\pi} \int_0^{\pi} \frac{\cos \theta}{x^2 + (x+y)^2 + y^2 - 2ax \sqrt{(x+y)^2 + y^2} \cos \theta} \frac{x}{\sqrt{(x+y)^2 + y^2}} \, d\theta$$

$$A_{y,j} = \frac{\mu_0 I_0}{4\pi} \int_0^{\pi} \frac{\cos \theta}{x^2 + (x+y)^2 + y^2 - 2ax \sqrt{(x+y)^2 + y^2} \cos \theta} \frac{y}{\sqrt{(x+y)^2 + y^2}} \, d\theta$$

Figure 2. Area of measurement of the pulsed magnetic field generated by the FOE coil. The origin of the coordinates is set at the center of the coil on the side in contact with the head. The x-axis is the long axis of the figure-of-8, and the y-axis is perpendicular to the x-axis. The square shaded area (120×100 mm) indicates the measurement area in the x-y plane. By moving the search coil by 5-mm increments in the x- and y-directions and 10-mm in the z-direction in a 3-D coordinate system, we measured magnetic flux densities at a total of 1575 sites (25×21×3).

and $a_j$ is the radius of the $j$-th loop $C_j$. The magnetic flux density can therefore be obtained by calculating the curl of the vector potential as described previously [16].

\[
B_x = (\nabla \times A) = \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y} \tag{8}
\]

\[
B_y = (\nabla \times A) = \frac{\partial A_z}{\partial z} - \frac{\partial A_x}{\partial y} \tag{9}
\]

\[
B_z = (\nabla \times A) = -\frac{\partial A_x}{\partial z} + \frac{\partial A_y}{\partial z} \tag{10}
\]

Figure 4 shows a lateral view of one wing of the FOE coil and the analysis planes 1 ($z = 10$ mm), 2 ($z = 20$ mm), and 3 ($z = 30$ mm). The size of the FOE coil (inner radius, 26 mm; outer radius, 44 mm; wire width, 1 mm; wire height, 7 mm) was based on the data from the literature [15]. The thickness of the FOE-coil including the plastic chassis was 30 mm. In this model the coil wire was located midway between the surfaces of the plastic chassis. The magnetic flux densities $B_j$ in the analysis planes 1, 2, and 3 were calculated as indicated above. Then, the theoretical values of the magnetic flux density were compared with the measured values.

Results

1. Relationship between CR values and CR integral magnetic flux density

As shown in Table 1, the larger the CR values, the nearer the CR integration values approached the value of numerical integration. However, the standard deviation increased as CR became larger ($5.99, 11.58$, and $21.36$ ms). Good agreement in magnetic flux density was obtained between the value obtained from numerical integration ($0.591$ T) and the values measured using the CR integration circuit at CR values of 4 to 6 ms. However, the standard deviation of CR integration was smaller for the CR value of 4.15 ms than for 5.99 ms. Thus, we selected CR = 4 ms.

2. Comparison of the values obtained by CR integration circuit and by numerical integration

Figure 5 shows the pulse shape of the digital values of magnetic flux density measurements displayed on a PC monitor. Peak magnetic flux density can be measured as the amplitude from baseline to peak of the wave.

Figure 6 shows a comparison of the magnetic flux density amplitudes measured by the CR integration circuit and those obtained by numerical integration of induced $V$, using the same search coil placed at the center of a circular coil with a diameter of 100 mm, as the applied voltage was changed from 400 to 2000 V.
The pulse width was kept constant at 0.2 ms throughout the measurements. The values obtained by CR integration circuit and numerical integration agreed well ($R^2 = 0.9993; P = 0.0000; \text{standard error} = 5.56 \times 10^{-3} \text{T}$). Similar good agreement was obtained for magnetic flux densities from 0.1 to 1.4 T and pulse widths from 0.1 to 0.6 ms (Table 2).

### Table 1. Relationship between CR values and magnetic flux density (T) measured by CR integration circuit.

<table>
<thead>
<tr>
<th>CR value (ms)</th>
<th>Measured magnetic flux density (T)</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.29</td>
<td>0.498</td>
<td>0.00</td>
<td>$9.40 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.60</td>
<td>0.545</td>
<td>9.40</td>
<td>$2.74 \times 10^{-3}$</td>
</tr>
<tr>
<td>1.45</td>
<td>0.563</td>
<td>1.10</td>
<td>$1.34 \times 10^{-3}$</td>
</tr>
<tr>
<td>2.88</td>
<td>0.581</td>
<td>1.10</td>
<td>$1.34 \times 10^{-3}$</td>
</tr>
<tr>
<td>4.15</td>
<td>0.585</td>
<td>1.45</td>
<td>$1.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>5.99</td>
<td>0.589</td>
<td>4.51</td>
<td>$1.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>11.58</td>
<td>0.578</td>
<td>5.00</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>21.36</td>
<td>0.654</td>
<td>5.00</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

C, capacitance ($9.705 \mu\text{F}$); R, resistance (30–2201 Ω); SD, standard deviation.

### Table 2. Comparison of magnetic flux densities measured by CR integration circuit and calculated by numerical integration, using the same search coil at various turns of the coil for different pulse widths and applied voltages.

<table>
<thead>
<tr>
<th>Number of turns</th>
<th>Applied voltage (V)</th>
<th>Pulse width (ms)</th>
<th>Output voltage (mV)</th>
<th>Magnetic flux density (T) By CR integration circuit</th>
<th>Magnetic flux density (T) By numerical integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
<td>0.118</td>
<td>7.7</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>1500</td>
<td>0.115</td>
<td>11.5</td>
<td>0.19</td>
<td>0.20</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>0.126</td>
<td>84</td>
<td>1.43</td>
<td>1.43</td>
</tr>
<tr>
<td>12</td>
<td>1000</td>
<td>0.237</td>
<td>29</td>
<td>0.51</td>
<td>0.49</td>
</tr>
<tr>
<td>20</td>
<td>1000</td>
<td>0.601</td>
<td>13</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>20</td>
<td>2000</td>
<td>0.613</td>
<td>26</td>
<td>0.44</td>
<td>0.44</td>
</tr>
</tbody>
</table>

**Figure 5.** A computer monitor image of a CR-integrated result. The horizontal axis indicates time with divisions of 0.1 ms, and the vertical axis indicates magnetic flux density with divisions of 0.1 T. Peak magnetic flux density can be measured as the amplitude from baseline to peak of the wave.

**Figure 6.** Comparison of magnetic flux densities measured by the CR integration circuit (horizontal axis) and the values obtained by numerical integration (vertical axis) using the same search coil with a pulse width of 0.2 ms. $R$, correlation coefficient; $Se$, standard error.

### 3. Magnetic flux density measured by CR integration circuit

The distribution of the horizontal and vertical magnetic flux densities near the FOE coil is shown in Fig. 7. There were two peaks for the horizontal magnetic flux density ($B_x$). The highest peak was over the center of the coil. At 100% output, $B_x$ at the three measured planes ($z = 10$, 20 and 30 mm) were 0.44, 0.37, and 0.22 T, respectively. At 50% output, $B_x$ at z...
Figure 7. Distribution of the magnetic flux density around a figure-of-eight coil. Horizontal element ($B_x$) and vertical element ($B_z$) at 100% and 50% output are shown for each measurement plane ($z = 10$, $20$ and $30$ mm).

There are two peaks for $B_x$; the higher peak is over the center of the coil. $B_z$ has one peak around the center of the circular wing of the coil. The magnetic flux density is higher at higher stimulation intensity, and lower at more distant sites from the coil.
Izumi S et al.: Integration circuit to measure pulsed magnetic field


10, 20 and 30 mm were 0.21, 0.14 and 0.09 T, respectively. The second peak was over the outer rim of the coil. At 100% output, $B_y$ at $z = 10$, 20 and 30 mm were 0.36, 0.29 and 0.14 T, respectively. At 50% output, at $z = 10$, 20 and 30 mm were 0.17, 0.10 and 0.07 T, respectively.

There was a peak for the vertical magnetic flux density ($B_z$), located around the center of the circular wing of the coil. At 100% output, $B_z$ at $z = 10$, 20 and 30 mm were 0.52, 0.38 and 0.23 T, respectively. At 50% output, $B_z$ at $z = 10$, 20 and 30 mm were 0.28, 0.19 and 0.18 T, respectively.

4. Magnetic flux density calculated by the FOE coil model

The theoretical values of the magnetic flux density in the FOE coil were compared with the actual measurements at 100% output power, for the transverse component $B_x$ along three representative test lines ($y' = 0; z = 10, 20,$ and 30 mm). The results are shown in Fig. 8. The calculated magnetic flux densities were approximately 1.8 times larger than the experimental values, but both results showed similar behaviors overall. As was expected from equation (8), $B_x$ had a peak at the center of the FOE coil, and showed secondary peaks at the edges of the FOE coil. The ratios of $B_x$ at the center to $B_x$ at the edge in the model were estimated to be 1.7, 2.1 and 2.5 at the heights of $z = 10, 20$ and 30 mm, respectively. These values were slightly larger than the measured values of 1.2, 1.3 and 1.6, respectively.

Discussion

We developed a search coil with a CR integration circuit for instantaneous measurement of pulsed magnetic fields, and confirmed the accuracy of the measurements by comparing the data obtained from the CR integration circuit with the values obtained from numerical integration. Although magnetic field measurement using CR integration is not a new idea, there are no reports on the appropriate CR values to use or measured magnetic flux densities generated by the FOE coil commonly used in clinical laboratories, probably because of the difficulties in manufacturing such measurement devices. One of the difficulties is the selection of optimal capacitance and resistance. The capacitance displayed on a capacitor is measured using direct current and includes errors. Furthermore, it varies depending on current frequency. Thus the optimal combination of capacitance and resistance in CR circuits to measure pulsed magnetic flux densities have never been clarified. We selected CR = 4 ms which proved to be appropriate for the measurement of magnetic field intensities ranging from 0.1 to 1.4 T with pulse widths ranging from 0.1 to 0.6 ms.

We report for the first time the distribution of magnetic flux densities generated by a FOE coil, which we measured using the CR integration circuit that we designed. For clinical application of TMS, it is essential to know the quality of the stimulators used in individual laboratories. Currently, the available specifications for such devices are limited to maximal $dB/dt$ and theoretical electric fields adjacent to the coil. Thus, the data presented in this study of actual magnetic flux densities measured at a distance of 1 to

Figure 8. Comparison of magnetic flux density parallel to the x-axis ($B_x$) between the model calculation and actual measurements at 100% output.

Data show $B_x$ along the lines $y' = 0$ and $z = 10, 20$ and 30 mm. In the graph of actual measurements, values of $B_x$ ($x < 0$) are plotted as mirror images of $B_x$ ($x > 0$). Calculated values and experimental ones are similar. The ratio of $B_x$ at the center (maximum peaks) to $B_x$ at the edge (secondary peaks) in the model was estimated to be 1.7, 2.1, 2.5 at the height of $z = 10, 20, 30$ mm, respectively. These quantities were slightly larger than the measured values of 1.2, 1.3, 1.6, respectively.
3 cm from the FOE coil surface at different stimulator outputs should be useful in quantitative analysis of the effects of TMS on cortical function, irrespective of the type of TMS device. The motor threshold of TMS is given in individual stimulators as relative values, and absolute values are not available. Okita and Takagi [17] studied the relation between magnetic field structure and distribution of induced electric current based on a cylindrical model composed of a uniform electrically conductive medium. By choosing representative parameters of conductivity (σ=0.25 S/m), frequency (f=2π×10^3 Hz) and scale of the medium (radius =10 cm) often used in TMS, they showed that the threshold of pulsed magnetic flux density for exciting motor nerve fibers was in the range of 0.5 – 0.8 T, which is reasonably consistent with the values reported by Izumi et al. [18]. Izumi et al. [18] found that the threshold values for activating the ulnar motor nerve at the elbow of a healthy subject ranged between 0.15 and 0.2 T at pulse widths of 0.1 to 0.4 ms. Since the motor threshold in healthy humans usually lies in the range of 50 to 100% output of the TMS device used in the present study, peak Bx amplitudes obtained from the present measurements suggest that the threshold of magnetic flux density in the human motor cortex is comparable to the threshold reported for motor nerve fibers.

A previous study that calculated the electric field near the surface of the FOE coil showed that the induced current flow is the strongest at the center of the coil, and is directed along the axis parallel to the coil handle, corresponding to the y-axis in the present study [15]. Thus, we compared the actual measurements and the model calculations of Bx, which determines Ex (induced electric field parallel to the y-axis).

Good agreement was observed between the model calculations and the actual measurements of Bx. In principle, the strength of the magnetic flux density is proportional to the electric current of the coil, which is assumed to be $I_c = 3$ kA according to the specification of the device. The differences between the theoretical and experimental values of Bx probably originated from this assumption. However, peak Bx amplitude at the coil center relative to that at the coil edge was smaller for the actual measurements than for the model calculations, suggesting less focalization of stimulation in the actual coil than in the model. This is probably because the model calculation assumes that the two loop currents touch each other at the center, while they do not in the actual coil [15].

Future studies should examine the relationship between coil geometry and the actual magnetic flux density as well as the induced electric current estimated from a medium model, in order to create an optimal coil design for TMS to be used in patients with various central nervous system disorders.

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